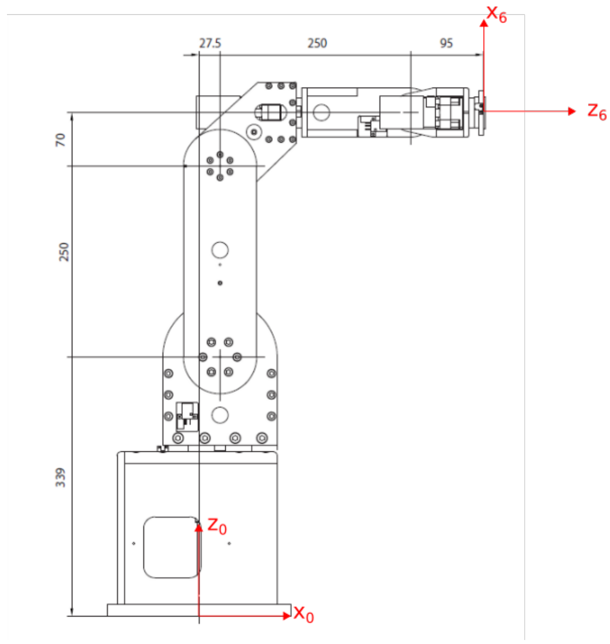
# ME 4/567 – ECE 564: Robotics and Automated Systems
# Spring 2025 — Project III: Robot-Arm Kinematics

In this project, we will solve the position and velocity level forward kinematics of the NeXCoM 6-DoF miniBoT and optionally verify that our solution matches with the output of its implementation in MuJoCo.

Question 1 .................................................... *60 (main) + 0 (bonus) points*

The figure on the right is a depiction of miniBOT6-R robot. Use the Denavit-Hartenberg (DH) convention to assign frames on each link. Furthermore, fill in the table on the left with DH parameters. A skeleton diagram of the robot is provided in Figure 1.

| | $\alpha$ | $a$ | $d$ | $\theta$ |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |




(a) [30 points] Solve the position-level forward kinematics problem. Use this solution to find the end-effector pose $\boldsymbol{\xi}$ given that the joint angles are

$$\boldsymbol{\theta} = \begin{bmatrix} 0^\circ & 90^\circ & 0^\circ & 0^\circ & -90^\circ & 0^\circ \end{bmatrix}.$$

Your end-effector $\boldsymbol{\xi}$ should be given by a 6-vector, the first three components of which is are the components of the translation vector from the base to the origin of the end-effector expressed in the base frame, and the last three of which are the EulerZYX angles of the end-effector frame with respect to the base frame.

(b) [30 points] Solve the position-level inverse kinematics of the problem in closed-form. Provide the expressions for the joint angles in terms of the given end-effector pose in your submission. Use this solution to find the joint angles whenever the end-effector pose is given by $\boldsymbol{\xi} = (\boldsymbol{R}, \boldsymbol{t})$, where

$$\boldsymbol{R} = \begin{bmatrix} 0.7551 & 0.4013 & 0.5184 \\ 0.6084 & -0.7235 & -0.3262 \\ 0.2441 & 0.5617 & -0.7905 \end{bmatrix}, \quad \boldsymbol{t} = \begin{bmatrix} 399.1255 \\ 171.01529 \\ 416.0308 \end{bmatrix}$$

Question 2 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . *40 (main) + 0 (bonus) points*

Compute the kinematic Jacobian for this manipulator that relates the joint angle rates to the end-effector velocity in closed-form. This expression can be performed without choosing a frame, as discussed in class.

Implement your Jacobian matrix in Python and solve for the joint angle rates needed to produce an end-effector spatial twist of

$$\boldsymbol{\nu} = \begin{pmatrix} \boldsymbol{\omega} & \boldsymbol{v} \end{pmatrix}^\top = \begin{pmatrix} 2 & -1 & 0.5 & 100 & -200 & -300 \end{pmatrix}^\top$$

whenever the manipulator is at a configuration given in question 1b.

Question 3 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . *20 (bonus) points*

Implement the miniBoT6-R in MuJoCo by imitating this example: [link], that performs a similar construction for a different robot arm, the Franka arm. You will have to replace the `xml` corresponding to the Franka arm with the one you will produce of the miniBoT6-R. You may have to convert rotation matrices to quaternions as needed as you proceed through the example.

Verify your solutions from questions 1 and 2 using functionality provided by MuJoCo, as done in the tutorial provided above.

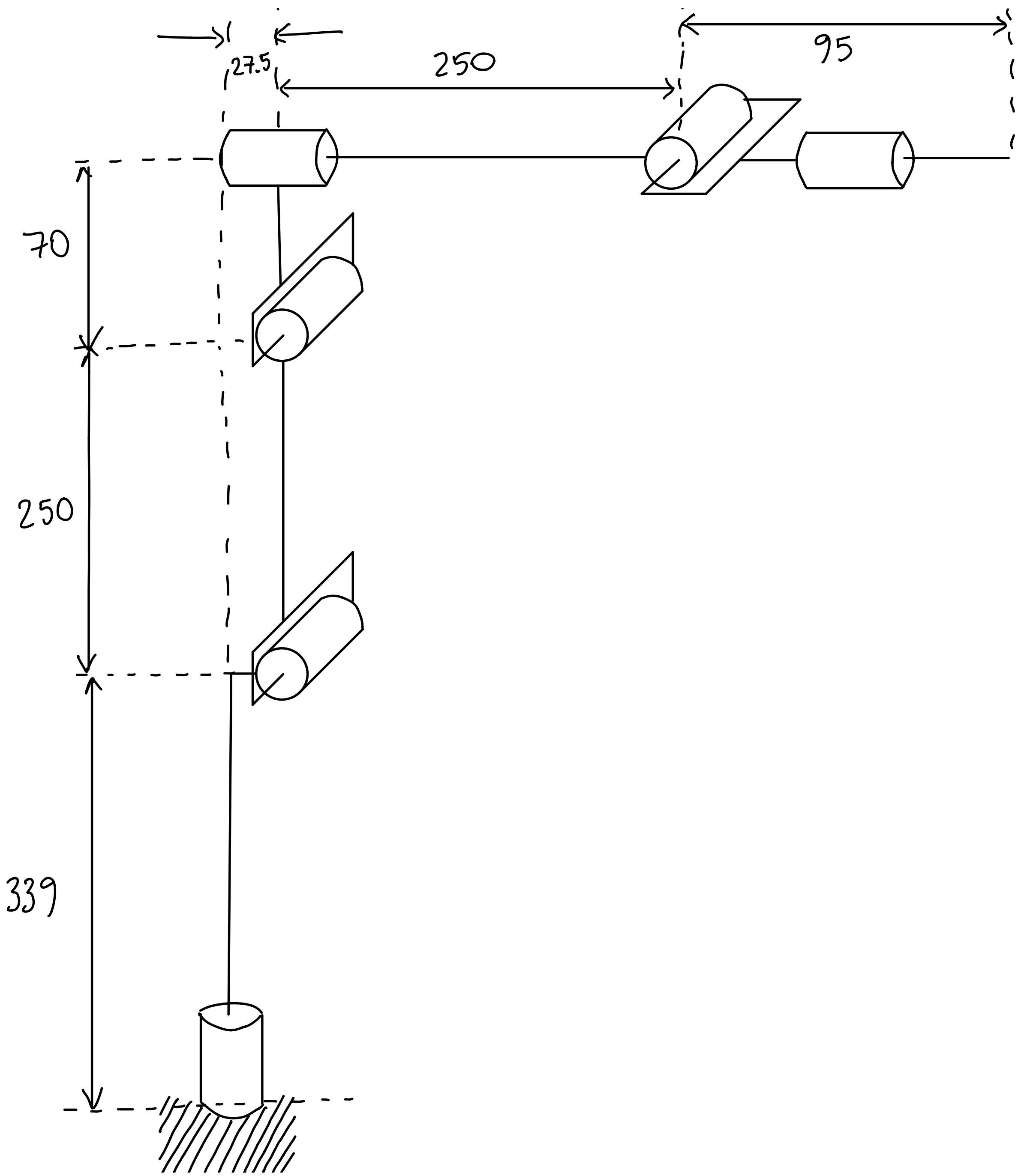



Figure 1: Skeleton diagram of the NeXCoM miniBOT-6